AFRL-RY-HS-TR-2010-0029

REMARKS ON THE RADIATIVE TRANSFER APPROACH TO SCATTERING OF ELECTROMAGNETIC WAVES IN LAYERED RANDOM MEDIA

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MARCH 2010
Final Report

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UNITED STATES AIR FORCE

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REPORT DOCUMENTATION PAGE

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1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
March 2010	Final	01 October 2006 – 30 September 2009

4. TITLE AND SUBTITLE

REMARKS ON THE RADIATIVE TRANSFER APPROACH TO SCATTERING OF ELECTROMAGNETIC WAVES IN LAYERED RANDOM MEDIA

5a. CONTRACT NUMBER
IN-HOUSE

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)

Saba Mudaliar

5d. PROJECT NUMBER
2304

5e. TASK NUMBER
HE

5f. WORK UNIT NUMBER
2304HE01

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Electromagnetics Technology Division
Electromagnetic Scattering Branch (AFRL/RYHE)
Sensors Directorate, Air Force Research Laboratory
Hanscom Air Force Base, MA 01731-2909
Air Force Materiel Command, United States Air Force

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-RY-HS-TR-2010-0029

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory
Sensors Directorate
Hanscom Air Force Base, MA 01731-2909
Air Force Materiel Command
United States Air Force

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/RYHE

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-RY-HS-TR-2010-0029

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
PAO Case Number: 66ABW-2009-0760; cleared 2009. This report contains the results from research and development sponsored and/or monitored by the Sensors Directorate at the Air Force Research Laboratory (AFRL) Hanscom Research Site, MA. The AFRL Hanscom Research Site was closed in 2012 as part of the Base Realignment and Closure Commission (BRAC) process. This report is the best available copy at time of publication.

14. ABSTRACT

The radiative transfer (RT) approach is widely used in applications involving scattering from layered random media with rough interfaces. Although it has been successful in several applications in various disciplines, it is well known that this approach involves certain approximations. In this document these assumptions and approximations are reexamined. To enable this, a statistical wave approach is employed to this problem and the governing equations for the first and second moments of the wave functions are derived. A transition is hence made to arrive at a system of equations corresponding to that of the RT approach. It is hence found that more conditions are implicitly involved in the RT approach than generally believed to be sufficient.

15. SUBJECT TERMS
random media, rough surfaces, radiative transfer, multilayer, wave approach

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	SAR	32	Saba Mudaliar
Unclassified	Unclassified	Unclassified			19b. TELEPHONE NUMBER *(Include Area Code)* N/A

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Table of Contents

List of Figures

1. Introduction

The model of layered random medium with rough interfaces is often encountered in many applications in various disciplines. A simple approach is to incoherently add the contributions of volumetric and surface fluctuations[36, 16]. However, this is valid only when we are in the single scattering regime[7, 20]. There are some other hybrid approaches [23, 6] which take into consideration some multiple scattering effects. Brown [3] outlines an iterative procedure which properly includes all multiple scattering interactions. However, it does not appear feasible to carry out the calculation beyond one or two iterations. Among the other methods currently used, perhaps the most widely used approach is the radiative transfer (RT) approach [31, 14, 13, 27, 4, 32, 17, 10]. Here one formulates the scattering and propagation in each layer by using the radiative transfer equation which involves only the parameters of the medium of that layer. The boundary conditions are derived separately and independently using some asymptotic procedure developed in rough surface scattering theory [2, 1, 33]. The RT equations, along with the boundary conditions, comprise the system that describes the problem.

Although this procedure appears to be reasonable and sound it is apparent that certain approximations are involved and we would like to know the conditions under which this kind of approach is appropriate. One way to better understand the RT approach is to compare it with the more rigorous wave approach. For the case of unbounded random media it was found [35] that the RT approach is applicable when (a) one uses the quasi-uniform field approximation, (b) one uses the ladder approximation to the intensity operator of the Bethe-Salpeter equation, and (c) the medium is statistically quasi-homogeneous.

However, our problem has bounded structures which are randomly rough. Therefore it remains to be seen whether the conditions arrived at in the case of unbounded random media will be sufficient for our problem.

In our project we employed a statistical wave approach using surface scattering operators [33, 21] to derive the transport equations for our multilayer problem. In this process we found that there are more conditions implied when we choose to apply the RT approach to our problem than it is widely believed to be necessary. One such condition is the weak surface correlation approximation. This means that the RT approach places certain restrictions on the type of rough interfaces that it can model accurately.

This report is organized as follows. In Section II we describe the geometry of the problem. In the next section we describe the RT approach to the problem. In Section IV we describe the wave approach to this problem. In Section V we transition to the transport equation system. The report concludes with a discussion of our findings.

2. Description of the Problem

The geometry of the problem is shown in Figure 1. We have an N-layer random medium stack with rough interfaces. The permittivity of the j-th layer is $\epsilon_j + \tilde{\epsilon}_j(\mathbf{r})$ where ϵ_j is the deterministic part and $\tilde{\epsilon}_j$ is the randomly fluctuating part. The permeability of all the layers is that of free space. The randomly rough interfaces are given as $z = z_j + \zeta_j(\mathbf{r}_\perp)$. It is assumed that $\tilde{\epsilon}_j$ and ζ_j are zero-mean isotropic stationary random processes independent of each other. Thus, on the average the interfaces are parallel planes. Let $z_0 = 0$, and let d_j be the thickness of the j-th layer. The media above and below the stack are homogeneous with parameters ϵ_0, k_0, and ϵ_{N+1}, k_{N+1}, respectively. This system is excited by a monochromatic electromagnetic plane wave and we are interested in formulating the resulting multiple scattering process.

3. Radiative Transfer Approach

Multiple scattering in a complex environment is well described by the radiative transfer theory. This theory is not only conceptually simple but also very efficient.
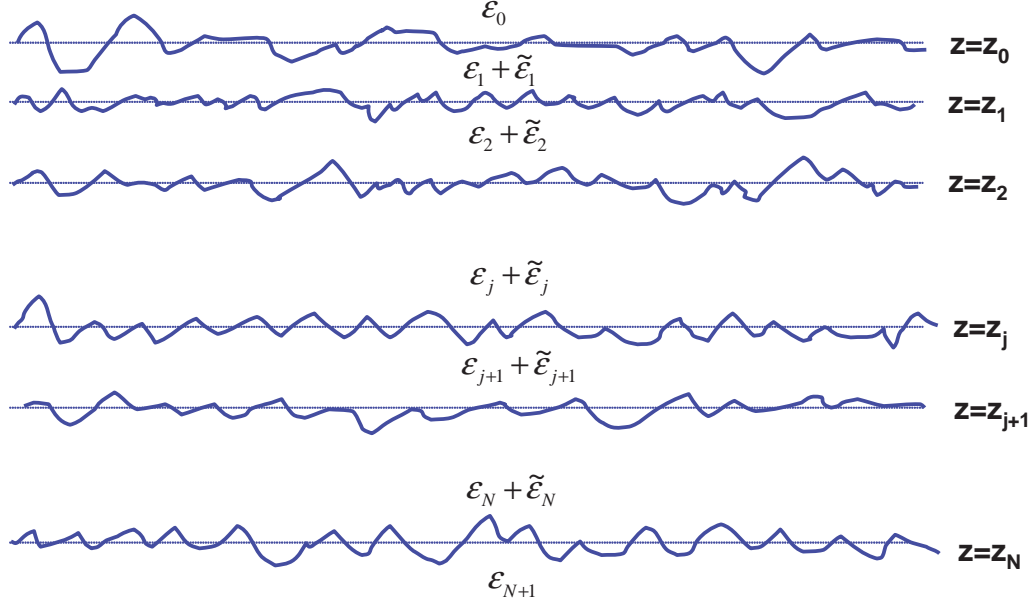
Figure 1: Geometry of the problem.

The fundamental quantity here is the specific intensity $\mathbf{I}$ which is governed by the following equation [5, 28, 12]

$$\hat{s} \cdot \nabla \mathbf{I}(\mathbf{r}, \hat{s}) + \bar{\bar{\gamma}}\, \mathbf{I}(\mathbf{r}, \hat{s}) = \int d\Omega' \bar{\mathbf{P}}(\hat{s}, \hat{s}') \mathbf{I}(\mathbf{r}, \hat{s}'). \tag{1}$$

One may regard this equation as a statement of conservation of energy density $\mathbf{I}$ which is a phase-space quantity at position $\mathbf{r}$ and direction $\hat{s}$. $\bar{\bar{\gamma}}$ is the extinction matrix which is a measure of loss of energy in direction $\hat{s}$ due to scattering in other directions. $\bar{\mathbf{P}}$ is the phase matrix representing increase in energy density in direction $\hat{s}$ due to scattering from neighbouring elements. Ω is the solid angle subtended by $\hat{s}$. Given the statistical characteristics of the medium one can readily calculate the phase matrix. The extinction matrix is hence calculated using the relation $\bar{\bar{\gamma}} = \int \bar{\mathbf{P}}(\hat{s}', \hat{s}) d\Omega'$. The specific intensity in each layer is governed by an equation similar to (1). Since our layer problem has translational invariance in azimuth the RT equation for the m-th layer takes the following form,

$$\cos\theta \frac{d}{dz} \mathbf{I}_m(z, \hat{s}) + \bar{\bar{\gamma}}_m\, \mathbf{I}_m(z, \hat{s}) = \int_{\Omega_m} d\Omega' \bar{\mathbf{P}}_m(\hat{s}, \hat{s}') \mathbf{I}_m(z, \hat{s}') \tag{2}$$

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3

where the subscript m denotes that the quantity corresponds to those of the m-th layer and θ is the elevation angle of $\hat{s}$. This set of RT equations is complemented by a set of boundary conditions which are in turn based on energy conservation considerations. To be more precise, we impose the condition that the energy flux density at each interface is conserved. This leads to the following boundary conditions on the m-th interface

$$\mathbf{I}_m^u(z_m, \hat{s}) = \int d\Omega' \, \langle \mathcal{R}_{m+1,m}(\hat{s}, \hat{s}') \rangle \, \mathbf{I}_m^d(z_m, \hat{s}') + \int d\Omega' \, \langle \mathcal{T}_{m,m+1}(\hat{s}, \hat{s}') \rangle \, \mathbf{I}_{m+1}^u(z_m, \hat{s}'). \quad (3)$$

The boundary conditions on the $(m-1)$-th interface are given as

$$\mathbf{I}_m^d(z_{m-1}, \hat{s}) = \int d\Omega' \, \langle \mathcal{R}_{m-1,m}(\hat{s}, \hat{s}') \rangle \, \mathbf{I}_m^u(z_{m-1}, \hat{s}') + \int d\Omega' \, \langle \mathcal{T}_{m,m-1}(\hat{s}, \hat{s}') \rangle \, \mathbf{I}_{m-1}^d(z_{m-1}, \hat{s}'), \tag{4}$$

where $\mathcal{R}_{mn}$ and $\mathcal{T}_{mn}$ are the local reflection and transmission Müller matrices. To be more specific, $\mathcal{R}_{mn}$ represents the reflection Müller matrix of waves incident from medium n on the interface that separates medium m and medium n. The superscripts u and d indicate whether the intensity corresponds to a wave travelling upwards or downwards. The integrations in these expressions are over a solid angle (hemisphere) corresponding to $\hat{s}'$. Suppose we have a plane wave incident on this stack from above. Then the downward travelling intensity in Region 0 is

$$\mathbf{I}_0^d(z, \hat{s}) = \mathbf{B}_0 \delta \left(\cos\theta_0 - \cos\theta_i \right) \delta(\phi_0 - \phi_i), \tag{5}$$

where $\mathbf{B}_0$ is the intensity of the incident plane wave and $\{\theta_i, \phi_i\}$ describes its direction. Since there is no source or scatterer in Region $N+1$,

$$\mathbf{I}_{N+1}^u(z, \hat{s}) = 0.$$

Notice again that these boundary conditions represent conservation of intensity at the interfaces.

We should point out that the radiative transfer approach as applied to a particular problem is only a model based on certain assumptions. Since the RT theory is used in

a variety of applications, the particular assumptions involved are described in terms of different terminologies, specific to the discipline where it is used. One good way to understand in more general terms the RT approach and the underlying assumptions is to compare it with the more rigorous statistical wave approach. For the case of an unbounded random medium this kind of study was carried out in the seventies [35]. From that study we learn that the radiative transfer theory can be applied under the following conditions:

1. Quasi-stationary field approximation.

2. Weak fluctuations:

   (a) Ladder approximation to the intensity operator.

   (b) Kraichnan approximation to the mass operator.

3. Statistical homogeneity of the medium fluctuations.

These are the well-known conditions that we associate with the RT approach. However, our problem has bounded structures and, further, they are randomly rough. The question is this: are the above conditions sufficient to apply the RT approach for our problem? This is the motivation for our investigation. We follow the statistical wave approach to this problem, derive the equations for the coherence functions, and hence make the transition to the RT equations. This procedure enables us to better understand the necessary conditions for using the RT approach for our problem.

4. Wave Approach

The following are the equations that govern the waves in the layer structure:

$$\nabla \times \nabla \times \mathbf{E}_j - k_j^2 \mathbf{E}_j = v_j \mathbf{E}_j \qquad j = 1, \cdots, N, \tag{6}$$

where $v_j \equiv \omega^2 \mu \tilde{\epsilon}_j(\mathbf{r})$ represents the volumetric fluctuation in Region j. For the homogeneous regions above and below we have

$$\nabla \times \nabla \times \mathbf{E}_0 - k_0^2 \mathbf{E}_0 = 0, \tag{7a}$$

$$\nabla \times \nabla \times \mathbf{E}_{N+1} - k_{N+1}^2 \mathbf{E}_{N+1} = 0. \tag{7b}$$

The boundary conditions at the j-th interface are

$$\hat{n} \times \mathbf{E}_j(\mathbf{r}_\perp, \zeta_j) = \hat{n} \times \mathbf{E}_{j+1}(\mathbf{r}_\perp, \zeta_j), \tag{8a}$$

$$\hat{n} \times \nabla \times \mathbf{E}_j(\mathbf{r}_\perp, \zeta_j) = \hat{n} \times \nabla \times \mathbf{E}_{j+1}(\mathbf{r}_\perp, \zeta_j). \tag{8b}$$

where $\hat{n}$ is the unit vector normal to the j-th interface with normal pointing into the medium j. This system is complemented by the radiation conditions well away from the stack. We assume that we know the solution to the problem without volumetric fluctuations, and denote it as $\check{\mathbf{E}}$. The corresponding Green's functions denoted as $\check{\bar{\mathbf{G}}}_{ij}$ are governed by the following set of equations:

$$\nabla \times \nabla \times \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}, \mathbf{r}') - k_j^2 \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}, \mathbf{r}') = \bar{\bar{\mathbf{I}}} \delta_{jk} \delta(\mathbf{r} - \mathbf{r}'), \tag{9a}$$

$$\hat{\mathbf{n}} \times \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}_\perp, \zeta_j; \mathbf{r}') = \hat{\mathbf{n}} \times \check{\bar{\mathbf{G}}}_{(j+1)k}(\mathbf{r}_\perp, \zeta_j; \mathbf{r}'), \tag{9b}$$

$$\hat{\mathbf{n}} \times \nabla \times \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}_\perp, \zeta_j; \mathbf{r}') = \hat{\mathbf{n}} \times \nabla \times \check{\bar{\mathbf{G}}}_{(j+1)k}(\mathbf{r}_\perp, \zeta_j; \mathbf{r}'). \tag{9c}$$

Another pair of equations similar to (9b) and (9c) corresponding to $(j-1)$-th interface need to be added to this list. $\bar{\bar{\mathbf{I}}}$ here is unit dyad. Using these Green's functions and the radiation conditions the wave functions can be represented as

$$\mathbf{E}_j(\mathbf{r}) = \check{\mathbf{E}}_j(\mathbf{r}) + \sum_{k=1}^{N} \int_{\Omega_k} d\mathbf{r}' \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}, \mathbf{r}') v_k(\mathbf{r}') \mathbf{E}_k(\mathbf{r}') \qquad j = 0, 1, \cdots, N+1 \tag{10}$$

where $\Omega_k = \{\mathbf{r}'_\perp; \zeta_k < z' < \zeta_{k-1}\}$. Note that $v_0 = v_{N+1} = 0$. We first average (10) w.r.t. volumetric fluctuations to get

$$\langle \mathbf{E}_j(\mathbf{r}) \rangle_v = \check{\mathbf{E}}_j(\mathbf{r}) +$$

$$+ \sum_{k=1}^{N} \sum_{l=1}^{N} \int_{\Omega_k} d\mathbf{r}' \int_{\Omega_l} d\mathbf{r}'' \check{\bar{\mathbf{G}}}_{jk}(\mathbf{r}, \mathbf{r}') \langle \bar{\mathbf{G}}_{kl}(\mathbf{r}', \mathbf{r}'') \rangle_v \langle v_k(\mathbf{r}') v_l(\mathbf{r}'') \rangle \langle \mathbf{E}_l(\mathbf{r}'') \rangle_v. \tag{11}$$

where $\bar{\mathbf{G}}_{kl}$ is governed by the following system of equations

$$\nabla \times \nabla \times \bar{\mathbf{G}}_{kl}(\mathbf{r}, \mathbf{r}') - k_k^2 \bar{\mathbf{G}}_{kl}(\mathbf{r}, \mathbf{r}') = \bar{\bar{\mathbf{I}}} \delta_{kl} \delta(\mathbf{r} - \mathbf{r}') + v_k \bar{\mathbf{G}}_{kl}(\mathbf{r}, \mathbf{r}')$$

$$\hat{\mathbf{n}} \times \bar{\mathbf{G}}_{kl}(\mathbf{r}_\perp, \zeta_k; \mathbf{r}') = \hat{\mathbf{n}} \times \bar{\mathbf{G}}_{(k+1)l}(\mathbf{r}_\perp, \zeta_k; \mathbf{r}'),$$

$$\hat{\mathbf{n}} \times \nabla \times \bar{\mathbf{G}}_{kl}(\mathbf{r}_\perp, \zeta_k; \mathbf{r}') = \hat{\mathbf{n}} \times \nabla \times \bar{\mathbf{G}}_{(k+1)l}(\mathbf{r}_\perp, \zeta_k; \mathbf{r}').$$

Here $\hat{n}$ being the unit vector normal to the k-th interface. We also have a similar set of boundary conditions on the $k-1$-th interface. The subscript v is used to denote averaging with respect to volumetric fluctuations. Here we have used a first order approximation to the mass operator based on weak fluctuations. The volumetric fluctuations in different regions are assumed to be uncorrelated, which means that

$$\langle v_k(\mathbf{r}')v_l(\mathbf{r}'')\rangle = \delta_{kl}C_k(\mathbf{r}' - \mathbf{r}''), \tag{12}$$

where C_k is the correlation function of the volumetric fluctuations in Region k. We have assumed that the fluctuations of the parameters of our problem are Gaussian and statistically homogeneous. Inserting (12) in (11) and employing $\nabla \times \nabla \times \bar{\bar{\mathbf{I}}} - k_j^2$ on (11) we get

$$\nabla \times \nabla \times \langle \mathbf{E}_j(\mathbf{r})\rangle_v - k_j^2 \langle \mathbf{E}_j(\mathbf{r})\rangle_v = \int_{\Omega_j} d\mathbf{r}' \langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}')\rangle_v C_j(\mathbf{r} - \mathbf{r}')\langle \mathbf{E}_j(\mathbf{r}')\rangle_v. \tag{13}$$

Next we average (13) over the surface fluctuations,

$$\nabla \times \nabla \times \langle \mathbf{E}_j(\mathbf{r})\rangle_{vs} - k_j^2 \langle \mathbf{E}_j(\mathbf{r})\rangle_{vs} = \int_{\bar{\Omega}_j} d\mathbf{r}' \left\langle \langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}')\rangle_v C_j(\mathbf{r} - \mathbf{r}')\langle \mathbf{E}_j(\mathbf{r}')\rangle_v \right\rangle_s$$

where the subscript s denotes averaging over surface fluctuations and $\bar{\Omega}_j = \{\mathbf{r}'_\perp; \hat{\mathbf{z}}_j < z' < \hat{\mathbf{z}}_{j-1}\}$. We approximate $\left\langle \langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}')\rangle_v C_j(\mathbf{r} - \mathbf{r}')\langle \mathbf{E}_j(\mathbf{r}')\rangle_v \right\rangle_s$ as $\langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}')\rangle_{vs} C_j(\mathbf{r} - \mathbf{r}')\langle \mathbf{E}_j(\mathbf{r}')\rangle_{vs}$ and obtain

$$\nabla \times \nabla \times \langle \mathbf{E}_j(\mathbf{r})\rangle_{vs} - k_j^2 \langle \mathbf{E}_j(\mathbf{r})\rangle_{vs} = \int_{\bar{\Omega}_j} d\mathbf{r}' \langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}')\rangle_{vs} C_j(\mathbf{r} - \mathbf{r}')\langle \mathbf{E}_j(\mathbf{r}')\rangle_{vs}. \tag{14}$$

We call this the weak surface correlation approximation. We will later find that this is one additional approximation necessary to arrive at the RT system. First note from (14) that $(\nabla \times \nabla \times \bar{\bar{\mathbf{I}}} - k_j^2)\langle \mathbf{E}_j(\mathbf{r})\rangle_{vs} = 0$ for $j = 0, N+1$. This means that the coherent propagation constants in regions above and below the layer stack are unaffected by the fluctuations of the problem. However, they indeed get modified within the stack

region. On writing (14) as $(\nabla \times \nabla \times \bar{\bar{\mathbf{I}}} - k_j^2 - \mathcal{L})\langle \psi_j \rangle = 0$, where $\mathcal{L}$ denotes the integral operator $\int_{\bar{\Omega}_j} d\mathbf{r}' \langle \bar{\mathbf{G}}_{jj}(\mathbf{r}, \mathbf{r}') \rangle_{vs} C_j(\mathbf{r} - \mathbf{r}')$, we infer that $\chi_j \equiv \sqrt{(k_j^2 + \mathcal{L})}$ represents the mean propagation constant in $\bar{\Omega}_j$. Observe that χ_j depends explicitly on the volumetric fluctuations in Region j and implicitly on the fluctuations of the stack, both volumetric and surface. This is in contrast to the RT approach where $\bar{\bar{\gamma}}_j$ depends exclusively on the volumetric fluctuations in Region j. Moreover, χ_j depends on the polarization if the fluctuations of the problem are anisotropic. Further, even if the volumetric fluctuations are isotropic χ_j will be polarization-dependent because of surface reflections. This is in contrast to the RT approach where $\bar{\bar{\gamma}}_j$ is polarization-dependent only when the volumetric fluctuations are anisotropic. Therefore the question is this: when do the effects of boundaries on the mean propagation constants become negligible? A first order solution to the above dispersion relation shows that in situations where the thickness of the layer is larger than the corresponding mean free path the influence of the boundaries on the mean propagation constants become negligible, as in the case of the RT system.

Since the problem is invariant under translations in azimuth the mean wave functions for our problem have the following form:

$$\langle \mathbf{E}_j^p(\mathbf{r}) \rangle_{vs} = \exp(i\mathbf{k}_{\perp i} \cdot \mathbf{r}) \left\{ A_j^p(\mathbf{k}_{\perp i})\mathbf{p}_j^+ \exp[iq_j^p z] + B_j^p(\mathbf{k}_{\perp i})\mathbf{p}_j^- \exp[-iq_j^p z] \right\}$$
$$j = 1, 2, \cdots, N, \qquad (15)$$

$$\langle \mathbf{E}_0^p(\mathbf{r}) \rangle_{vs} = \exp(i\mathbf{k}_{\perp i} \cdot \mathbf{r}) \left\{ \mathbf{p}_0^- \exp[-ik_{0zi}z] + R^p(\mathbf{k}_{\perp i})\mathbf{p}_0^+ \exp[ik_{0zi}z] \right\}, \qquad (16)$$

and

$$\langle \mathbf{E}_{N+1}^p(\mathbf{r}) \rangle_{vs} = \exp(i\mathbf{k}_{\perp i} \cdot \mathbf{r})T^p(\mathbf{k}_{\perp i})\mathbf{p}_{N+1}^- \exp[-ik_{(N+1)zi}z], \qquad (17)$$

where the superscript p stands for the polarization, either horizontal or vertical. $\mathbf{p}$ is the unit vector representing polarization. q_j is the z-component of $\boldsymbol{\chi}_j$. The subscript i is used to indicate that the wave vector is in the incident direction. R and T denote, respectively, the mean reflection and transmission coefficient of the stack. A_j and B_j

denote, respectively, the mean coefficients of up-going and down-going waves in the j-th layer. Based on this we can formulate the waves averaged w.r.t. volumetric fluctuations as

$$\langle \mathbf{E}_j(\mathbf{r})\rangle_v^p = \frac{1}{4\pi^2}\int d\mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r})\Big\{A_j^{pq}(\mathbf{k}_\perp, \mathbf{k}_{\perp i})\mathbf{q}_j^+ \exp[iq_j z] + B_j^{pq}(\mathbf{k}_\perp, \mathbf{k}_{\perp i})\mathbf{q}_j^- \exp[-iq_j z]\Big\}$$
$$j = 1, 2, \cdots, N, \qquad (18)$$

$$\langle \mathbf{E}_0(\mathbf{r})\rangle_v^p = \exp(i\mathbf{k}_{\perp i}\cdot \mathbf{r})\exp[-ik_{0zi}z]\mathbf{p}_0^- +$$
$$+\frac{1}{4\pi^2}\int d\mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r})R^{pq}(\mathbf{k}_\perp, \mathbf{k}_{\perp i})\mathbf{q}_0^+ \exp[ik_{0z}z], \qquad (19)$$

and

$$\langle \mathbf{E}_{N+1}(\mathbf{r})\rangle_v^p = \frac{1}{4\pi^2}\int d\mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r})T^{pq}(\mathbf{k}_\perp, \mathbf{k}_{\perp i})\mathbf{q}_{N+1}^- \exp[-ik_{(N+1)z}z]. \qquad (20)$$

where A_j, B_j, R and T are now integral operators representing scattering from rough interfaces. The boundary conditions associated with the above equations at the j-th interface are

$$\hat{\mathbf{n}} \times \langle \mathbf{E}_j(\mathbf{r}_\perp, \zeta_j)\rangle_v = \hat{\mathbf{n}} \times \langle \mathbf{E}_{j+1}(\mathbf{r}_\perp, \zeta_j)\rangle_v \qquad j = 1, 2, \cdots, N \qquad (21a)$$

and

$$\hat{\mathbf{n}} \times \nabla \times \langle \mathbf{E}_j(\mathbf{r}_\perp, \zeta_j)\rangle_v = \hat{\mathbf{n}} \times \nabla \times \langle \mathbf{E}_{j+1}(\mathbf{r}_\perp, \zeta_j)\rangle_v \qquad j = 1, 2, \cdots, N. \qquad (21b)$$

The above system may be solved either numerically or by anyone of the asymptotic methods* available in rough surface scattering theory [2, 1, 33] to evaluate the mean coefficients that appear in (15)-(17).

We proceed now to the analysis of the second moments, by starting with (10). For convenience we write it in symbolic form as

$$\mathbf{E}_j = \check{\mathbf{E}}_j + \sum_{k=1}^{N}\check{\mathbf{G}}_{jk}v_k\mathbf{E}_k. \qquad (22)$$

We take the tensor product of this equation with its complex conjugate and average w.r.t. volumetric fluctuations and obtain

$$\big\langle \mathbf{E}_j \otimes \mathbf{E}_j^*\big\rangle_v = \langle \mathbf{E}_j\rangle_v \otimes \langle \mathbf{E}_j^*\rangle_v + \sum_{k=1}^{N}\sum_{k'=1}^{N}\sum_{l=1}^{N}\sum_{l'=1}^{N}\langle \bar{\mathbf{G}}_{jk}\rangle_v \otimes \langle \bar{\mathbf{G}}_{jk'}^*\rangle_v \hat{\mathbf{K}}_{kk'll'}\big\langle \mathbf{E}_l \otimes \mathbf{E}_{l'}^*\big\rangle_v, \quad (23)$$

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*It is necessary, however, to meet the weak surface correlation approximation employed earlier

where $\mathbf{K}$ is the intensity operator of the volumetric fluctuations. Employing the weak fluctuation approximation we approximate $\mathbf{K}$ by its leading term

$$\hat{\mathbf{K}}_{kk'll'} \simeq \langle v_k \otimes v_k^* \rangle \delta_{kk'll'} \bar{\bar{\mathbf{I}}} . \tag{24}$$

Next we average (23) w.r.t. the surface fluctuations and employ the weak surface correlation approximation, as before, to get

$$\left\langle \mathbf{E}_j \otimes \mathbf{E}_j^* \right\rangle_{vs} = \left\langle \langle \mathbf{E}_j \rangle_v \otimes \langle \mathbf{E}_j^* \rangle_v \right\rangle_s + \sum_{k=1}^{N} \left\langle \langle \bar{\mathbf{G}}_{jk} \rangle_v \otimes \langle \bar{\mathbf{G}}_{jk}^* \rangle_v \right\rangle_s \langle v_k \otimes v_k^* \rangle \left\langle \mathbf{E}_k \otimes \mathbf{E}_k^* \right\rangle_{vs} . \tag{25}$$

The above is an equation for the second moment of the wave function $\mathbf{E}$, which can be decomposed into a coherent part $\bar{\mathbf{E}}$ and a diffuse part $\tilde{\mathbf{E}}$. Therefore,

$$\langle \mathbf{E} \otimes \mathbf{E}^* \rangle = \langle \mathbf{E} \rangle \otimes \langle \mathbf{E}^* \rangle + \langle \tilde{\mathbf{E}} \otimes \tilde{\mathbf{E}}^* \rangle. \tag{26}$$

The coherent part is not of much interest; we know that it is specular for our problem. The diffuse or the incoherent part is of more interest. Therefore we write (25) in terms of diffuse fields:

$$\left\langle \tilde{\mathbf{E}}_j \otimes \tilde{\mathbf{E}}_j^* \right\rangle = \left\langle \widetilde{\langle \mathbf{E}_j \rangle}_v \otimes \widetilde{\langle \mathbf{E}_j^* \rangle}_v \right\rangle_s + \sum_{k=1}^{N} \left\langle \langle \bar{\mathbf{G}}_{jk} \rangle_v \otimes \langle \bar{\mathbf{G}}_{jk}^* \rangle_v \right\rangle_s \langle v_k \otimes v_k^* \rangle \left\langle \mathbf{E}_k \otimes \mathbf{E}_k^* \right\rangle_{vs} , \tag{27}$$

where $\widetilde{\langle \mathbf{E}_j \rangle}_v$ is the fluctuating part of $\langle \mathbf{E}_j \rangle_v$. Let us now write (27) in more detail as:

$$\left\langle \tilde{\mathbf{E}}_j(\mathbf{r}) \otimes \tilde{\mathbf{E}}_j^*(\mathbf{r}') \right\rangle = \left\langle \widetilde{\langle \mathbf{E}_j(\mathbf{r}) \rangle}_v \otimes \widetilde{\langle \mathbf{E}_j^*(\mathbf{r}') \rangle}_v \right\rangle_s + \sum_{k=1}^{N} \int_{\bar{\Omega}_k} d\mathbf{r}_1 \int_{\bar{\Omega}_k} d\mathbf{r}_1'$$

$$\left\langle \langle \bar{\mathbf{G}}_{jk}(\mathbf{r}, \mathbf{r}_1) \rangle_v \otimes \langle \bar{\mathbf{G}}_{jk}^*(\mathbf{r}', \mathbf{r}_1') \rangle_v \right\rangle_s |k_k|^4 C_k(\mathbf{r}_1 - \mathbf{r}_1') \rangle \left\langle \mathbf{E}_k(\mathbf{r}_1) \otimes \mathbf{E}_k^*(\mathbf{r}_1') \right\rangle_{vs} . \tag{28}$$

As it stands this equation is not convenient for seeking solution either analytically or numerically. Besides, one important goal for us is to investigate the conditions needed for employing the radiative transfer approach. With this in mind we introduce Wigner transforms. Note that (28) is an equation for the coherence function. On the other hand the RT equation, as we saw earlier, is an equation for the specific intensity which is a 'phase-space' quantity. Wigner transforms serve as a bridge to link these two quantities [34, 9, 18, 24].

We introduce Wigner transforms of waves and Green's functions as

$$\mathcal{E}_m\left(\frac{\mathbf{r}+\mathbf{r}'}{2},\mathbf{k}\right) = \int d(\mathbf{r}-\mathbf{r}')\,\langle \mathbf{E}_m(\mathbf{r})\otimes\mathbf{E}_m^*(\mathbf{r}')\rangle\,e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \tag{29}$$

$$\mathcal{G}_{mn}\left(\frac{\mathbf{r}+\mathbf{r}'}{2},\mathbf{k}\Big|\frac{\mathbf{r}_1+\mathbf{r}_1'}{2},\mathbf{l}\right) = \int d(\mathbf{r}-\mathbf{r}')\int d(\mathbf{r}_1-\mathbf{r}_1')$$
$$e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}e^{i\mathbf{l}\cdot(\mathbf{r}_1-\mathbf{r}_1')}\Big\langle\,\langle\bar{\mathbf{G}}_{mn}(\mathbf{r},\mathbf{r}_1)\rangle_v\otimes\langle\bar{\mathbf{G}}_{mn}^*(\mathbf{r}',\mathbf{r}_1')\rangle_v\,\Big\rangle_s. \tag{30}$$

In terms of these transforms (28) becomes

$$\tilde{\mathcal{E}}_m(\mathbf{r},\mathbf{k}) = \tilde{\mathcal{E}}_m^s(\mathbf{r},\mathbf{k}) + \frac{1}{(2\pi)^6}\sum_{n=1}^N |k_n|^4 \int_{\bar{\Omega}_n} d\mathbf{r}'\int d\boldsymbol{\alpha}\int d\boldsymbol{\beta}\,\mathcal{G}_{mn}(\mathbf{r},\mathbf{k}|\mathbf{r}',\boldsymbol{\alpha})\Phi_n(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_n(\mathbf{r}',\boldsymbol{\beta}), \tag{31}$$

where Φ_n is the spectral density of the volumetric fluctuations in the n-th layer. We have used the superscript s in the first term to indicate that this is due to surface scattering as defined by the first term in (28).

The fact that our problem has translational invariance in azimuth implies the following:

$$\mathcal{E}_m(\mathbf{r},\mathbf{k}) = \mathcal{E}_m(z,\mathbf{k}), \tag{32a}$$

$$\mathcal{G}_{mn}(\mathbf{r},\mathbf{k}|\mathbf{r}',\mathbf{l}) = \mathcal{G}_{mn}(z,\mathbf{k}|z',\mathbf{l};\mathbf{r}_\perp-\mathbf{r}'_\perp). \tag{32b}$$

Using these relations in (31) we have

$$\tilde{\mathcal{E}}_m(z,\mathbf{k}) = \tilde{\mathcal{E}}_m^s(z,\mathbf{k}) + \frac{1}{(2\pi)^6}\sum_{n=1}^N |k_n|^4 \int_{z_n}^{z_{n-1}} dz'\int d\boldsymbol{\alpha}\int d\boldsymbol{\beta}$$
$$\mathcal{G}_{mn}(z,\mathbf{k}|z',\boldsymbol{\alpha};0)\Phi_n(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_n(z',\boldsymbol{\beta}), \tag{33}$$

where $\mathcal{G}_{mn}(z,\mathbf{k}|z',\boldsymbol{\alpha};0)$ is the Fourier transform of $\mathcal{G}_{mn}(z,\mathbf{k}|z',\boldsymbol{\alpha};\mathbf{r}_\perp-\mathbf{r}'_\perp)$ w.r.t. $\mathbf{r}_\perp-\mathbf{r}'_\perp$ evaluated at the origin of the spectral space. To proceed further we need to evaluate $\mathcal{G}_{mn}$. Furthermore, we need to relate this system with that of RT, which involves the boundary conditions at the interfaces. In view of this we need to identify the coherence functions corresponding to up- and down-going wave functions. To facilitate this we decompose $\langle G_{mn}\rangle_v$ into its components,

$$\langle\bar{\mathbf{G}}_{mn}\rangle_v = \delta_{mn}\bar{\mathbf{G}}_m^o + \bar{\mathbf{G}}_{mn}^{uu} + \bar{\mathbf{G}}_{mn}^{ud} + \bar{\mathbf{G}}_{mn}^{du} + \bar{\mathbf{G}}_{mn}^{dd}, \tag{34}$$

11

where the first term is the singular part of the Green's function. The superscripts u and d indicate up- and down-going elements of the waves. The other components are due to reflections from boundaries. These are formally constructed using the concept of surface scattering operators as follows [33],

$$\left\langle G_{mn}^{ab}(\mathbf{r}, \mathbf{r}') \right\rangle_v^{\mu\nu} =$$

$$= \frac{1}{(2\pi)^4} \int d\mathbf{k}_\perp \int d\mathbf{k}'_\perp \{S_{mn}^{ab}(\mathbf{k}_\perp, \mathbf{k}'_\perp)\}^{\mu\nu} e^{i\mathbf{k}_\perp \cdot \mathbf{r} + iaq_m^\mu(\mathbf{k}_\perp)z} e^{-i\mathbf{k}'_\perp \cdot \mathbf{r}' - ibq_n^\nu(\mathbf{k}'_\perp)z'}, \quad (35)$$

where $\bar{\mathbf{S}}_{mn}^{ab}$ is the surface scattering operator. The superscripts a and b on S are used to indicate whether the waves are up-going or down-going. In the exponents, $a, b = 1$ if the waves are up-going. We let $a, b = -1$ if the waves are down-going. The z-component of the mean propagation constants in the n-th layer is denoted as q_n.

We recall that $\mathcal{G}_{mn}$ is the Wigner transform of $\left\langle \left\langle \bar{\mathbf{G}}_{mn} \right\rangle_v \otimes \left\langle \bar{\mathbf{G}}_{mn}^* \right\rangle_v \right\rangle_s$. The superscripts μ, ν stand for polarization, either h or v. It is important to note that only the in the quasi-uniform limit does the Wigner transform of the coherence function lead to the specific intensity of the RT equation. For our layer geometry the Green's function is nonuniform. However, each of its components given in (34) is quasi-uniform. When we use (34) to perform the Wigner transform we ignore all cross terms. In other words, we make the following approximation,

$$\mathcal{G}_{mn} \simeq \delta_{mn} \mathcal{G}_m^o + \mathcal{G}_{mn}^{uu} + \mathcal{G}_{mn}^{ud} + \mathcal{G}_{mn}^{du} + \mathcal{G}_{mn}^{dd},$$

where $\mathcal{G}_{mn}^{ab}$ is the Wigner transform of $\left\langle \left\langle \bar{\mathbf{G}}_{mn}^{ab} \right\rangle_v \otimes \left\langle \bar{\mathbf{G}}_{mn}^{ab*} \right\rangle_v \right\rangle_s$. Most of the cross terms are nonuniform and may be neglected under the quasi-uniform field assumption. A couple of cross terms are quasi-uniform and their inclusion lead to phase matrices that are different from those of the RT system. It turns out that such additional coherence terms become negligible when the layer thickness is of the same order or greater than the mean free path of the corresponding layer. It is under these conditions the approximate expression for $\mathcal{G}_{mn}$ given above is good.

With the introduction of this representation for $\mathcal{G}_{mn}$ in (31) we can trace up- and

down-going waves to obtain the following equations for the coherence function:

$$\tilde{\mathcal{E}}_m^u(z,\mathbf{k}) = \tilde{\mathcal{E}}_m^{su}(z,\mathbf{k}) +$$

$$+\frac{1}{(2\pi)^6}|k_m|^4 \int_{z_m}^z dz' \int d\boldsymbol{\alpha} \int d\boldsymbol{\beta} \ \mathcal{G}_m^>(z,\mathbf{k}|z',\boldsymbol{\alpha};0)\Phi_m(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_m(z',\boldsymbol{\beta})$$

$$+\frac{1}{(2\pi)^6}\sum_{n=1}^N |k_n|^4 \int_{z_n}^{z_{n-1}} dz' \int d\boldsymbol{\alpha} \int d\boldsymbol{\beta} \ \mathcal{G}_{mn}^{ua}(z,\mathbf{k}|z',\boldsymbol{\alpha};0)\Phi_n(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_n^a(z',\boldsymbol{\beta})$$

$$(36a)$$

$$\tilde{\mathcal{E}}_m^d(z,\mathbf{k}) = \tilde{\mathcal{E}}_m^{sd}(z,\mathbf{k}) +$$

$$+\frac{1}{(2\pi)^6}|k_m|^4 \int_z^{z_{m-1}} dz' \int d\boldsymbol{\alpha} \int d\boldsymbol{\beta} \ \mathcal{G}_m^<(z,\mathbf{k}|z',\boldsymbol{\alpha};0)\Phi_m(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_m(z',\boldsymbol{\beta})$$

$$+\frac{1}{(2\pi)^6}\sum_{n=1}^N |k_n|^4 \int_{z_n}^{z_{n-1}} dz' \int d\boldsymbol{\alpha} \int d\boldsymbol{\beta} \ \mathcal{G}_{mn}^{da}(z,\mathbf{k}|z',\boldsymbol{\alpha};0)\Phi_n(\boldsymbol{\alpha}-\boldsymbol{\beta})\mathcal{E}_n^a(z',\boldsymbol{\beta})$$

$$(36b)$$

Note that summation over $a = \{u,d\}$ is implied in the above equations. The first term in these equations, $\tilde{\mathcal{E}}^{sa}$, represents the contribution due exclusively to surface scattering, and has the following form:

$$\left\{\tilde{\mathcal{E}}_m^{sa}(z,\mathbf{k})\right\}^{\mu\nu} = 2\pi\delta\left\{k_z - \frac{1}{2}a\left[q_m^\mu(\mathbf{k}_\perp) + q_m^{\nu*}(\mathbf{k}_\perp)\right]\right\}$$

$$e^{ia[q_m^\mu - q_m^{\nu*}]z}\left\langle \left\{\tilde{\Sigma}_m^a\right\}^{\mu\mu'}\left\{\tilde{\Sigma}_m^{a*}\right\}^{\nu\nu'}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})\right\rangle_s E_{\mu'i}E_{\nu'i}^*, \qquad (37)$$

where Σ_m^a is the amplitude of the up-going wave in the m-th layer after volumetric averaging is performed. This means that it is a random function of surface fluctuations. When we substitute (37) and the expressions for $\mathcal{G}_{mn}$ in (36) we find that

$$\left\{\tilde{\mathcal{E}}_m^a(z,\mathbf{k})\right\}_{\mu\nu} = 2\pi\delta\left\{k_z - \frac{1}{2}a[q_m^\mu(\mathbf{k}_\perp) + q_m^{\nu*}(\mathbf{k}_\perp)]\right\}e^{ia[q_m^\mu - q_m^{\nu*}]z}\left\{\tilde{\mathcal{E}}_m^a(z,\mathbf{k}_\perp)\right\}_{\mu\nu}. \quad (38)$$

On substituting this in (36) and differentiating w.r.t. z we obtain the following transport equations:

$$\left\{\frac{d}{dz} - i\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right]\right\}\tilde{\mathcal{E}}_{\mu\nu}^u(z,\mathbf{k}_\perp) = \tilde{\mathcal{E}}_{\mu\nu}^u(z,\mathbf{k}_\perp) + \frac{|k_m|^4}{(2\pi)^2}\int d\boldsymbol{\alpha}_\perp S_\mu^> S_\nu^{>*}$$

$$\Phi_m\left\{\mathbf{k}_\perp - \boldsymbol{\alpha}_\perp;\frac{1}{2}\left[q_\mu(\mathbf{k}_\perp) + q_\nu^*(\mathbf{k}_\perp)\right] - \frac{1}{2}a\left[q_{\mu'}(\boldsymbol{\alpha}_\perp) + q_{\nu'}^*(\boldsymbol{\alpha}_\perp)\right]\right\}(\boldsymbol{\mu}\cdot\boldsymbol{\mu}')(\boldsymbol{\nu}\cdot\boldsymbol{\nu}')\tilde{\mathcal{E}}_{\mu'\nu'}^a(z,\boldsymbol{\alpha}_\perp),$$

$$(39a)$$

$$\left\{ -\frac{d}{dz} - i\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right] \right\} \tilde{\mathcal{E}}_{\mu\nu}^d(z, \mathbf{k}_\perp) = \tilde{\tilde{\mathcal{E}}}_{\mu\nu}^d(z, \mathbf{k}_\perp) + \frac{|k_m|^4}{(2\pi)^2} \int d\boldsymbol{\alpha}_\perp S_\mu^< S_\nu^{<*}$$

$$\Phi_m \left\{ \mathbf{k}_\perp - \boldsymbol{\alpha}_\perp; -\frac{1}{2}\left[q_\mu(\mathbf{k}_\perp) + q_\nu^*(\mathbf{k}_\perp)\right] - \frac{1}{2}a\left[q_{\mu'}(\boldsymbol{\alpha}_\perp) + q_{\nu'}^*(\boldsymbol{\alpha}_\perp)\right] \right\} (\boldsymbol{\mu}\cdot\boldsymbol{\mu}')(\boldsymbol{\nu}\cdot\boldsymbol{\nu}')\tilde{\mathcal{E}}_{\mu'\nu'}^a(z, \boldsymbol{\alpha}_\perp),$$

$$(39b)$$

where $\tilde{\tilde{\mathcal{E}}}_{\mu\nu}^a$ represents scattering due to the coherent part of $\mathcal{E}$, whereas the integral term in (39) represents scattering due to the diffuse part of $\mathcal{E}$. We may also regard $\tilde{\tilde{\mathcal{E}}}_{\mu\nu}^a$ as the source to our transport equations; it is given as

$$\tilde{\tilde{\mathcal{E}}}_{\mu\nu}^a = |k_m|^4 \, \Phi_m \left\{ \mathbf{k}_\perp - \mathbf{k}_{\perp i}; \frac{1}{2}a\left[q_\mu(\mathbf{k}_\perp) + q_\nu^*(\mathbf{k}_\perp)\right] - \frac{1}{2}b\left[q_\mu(\mathbf{k}_{\perp i}) + q_\nu^*(\mathbf{k}_{\perp i})\right] \right\}$$

$$\left(S_\mu^a S_\mu^{a*} \right)(\boldsymbol{\mu} \cdot \boldsymbol{\mu}_i)(\boldsymbol{\nu} \cdot \boldsymbol{\nu}_i)\left(\langle \mathcal{S}_{m0}^b \rangle_{\mu i} E_{\mu i} \right)\left(\langle \mathcal{S}_{m0}^b \rangle_{\nu i} E_{\nu i} \right)^*. \tag{40}$$

where summation over b is implied. Note that $\tilde{\mathcal{E}}^a$ in (39) includes both $\tilde{\mathcal{E}}^u$ and $\tilde{\mathcal{E}}^d$ (corresponding to up- and down-going waves). When the superscripts $\{a, b\}$ correspond to u, the value of $\{a, b\}$ in the argument of Φ_m takes the value $+1$; on the other hand, when the superscripts $\{a, b\}$ correspond to d the value of a in the argument of Φ_m takes the value -1. Since all quantities in (39) and (40) correspond to the same layer m we have dropped the subscript m to avoid cumbersome notations. Summation over μ' and ν' is implicit in (39). Similarly, summation over μ_i and ν_i is implicit in (40). $\mathcal{S}_{m0}^b$ is the scattering amplitude of waves with direction b in m-th layer due to wave incident in Region 0. To obtain appropriate boundary conditions we have to go back to the integral equation representations for $\tilde{\mathcal{E}}_{\mu\nu}^u$ and $\tilde{\mathcal{E}}_{\mu\nu}^d$ and observe their behaviour at the interfaces and try to find a relation between them.

After some manipulations we arrived at the following boundary conditions. At the $(m-1)$-th interface we have

$$\tilde{\mathcal{E}}_m^d(z_{m-1}, \mathbf{k}_\perp) = \int d\mathbf{k}_\perp' \left\langle \dddot{\mathcal{R}}_{m-1,m}\left(\mathbf{k}_\perp, \mathbf{k}_\perp'\right) \right\rangle \tilde{\mathcal{E}}_m^u(z_{m-1}, \mathbf{k}_\perp'), \tag{41a}$$

with $\dddot{\mathcal{R}} = \dddot{\mathbf{R}} \otimes \dddot{\mathbf{R}}^*$ where $\dddot{\mathbf{R}}_{m-1,m}$ is the stack reflection matrix (not the local reflection matrix) for a wave incident from below on the $(m-1)$-th interface. Similarly

$$\tilde{\mathcal{E}}_m^u(z_{m-1}, \mathbf{k}_\perp) = \int d\mathbf{k}_\perp' \left\langle \dddot{\mathcal{R}}_{m+1,m}\left(\mathbf{k}_\perp, \mathbf{k}_\perp'\right) \right\rangle \tilde{\mathcal{E}}_m^d(z_{m-1}, \mathbf{k}_\perp'), \tag{41b}$$

where $\overset{...}{\mathcal{R}}_{m+1,m}$ is the tensor product of stack reflection matrix for a wave incident from above on the m-th interface.

We were able to obtain the boundary conditions only after imposing certain approximations as given below. Consider the following identity:

$$\bar{\mathbf{S}}_{mm}^{du} = \bar{\mathbf{F}}_m \overset{...}{\bar{\mathbf{R}}}_{m-1,m} \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\} \bar{\mathbf{F}}_m \tag{42}$$

where $\bar{\mathbf{F}}_m = \text{diag} \left\{ e^{iq_h d_m}, e^{iq_v d_m} \right\}$. Notice that this is an operator relation where all elements are operators. Taking the tensor product of (42) with its complex conjugate we have

$$\bar{\mathbf{S}}_{mm}^{du} \otimes \bar{\mathbf{S}}_{mm}^{du*} = \left(\bar{\mathbf{F}}_m \otimes \bar{\mathbf{F}}_m^* \right) \left(\overset{...}{\bar{\mathbf{R}}}_{m-1,m} \otimes \overset{...}{\bar{\mathbf{R}}}_{m-1,m}^* \right) \left(\left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\} \otimes \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\}^* \right) \left(\bar{\mathbf{F}}_m \otimes \bar{\mathbf{F}}_m^* \right). \tag{43}$$

Next we average (43) w.r.t. surface fluctuations and get

$$\left\langle \bar{\mathbf{S}}_{mm}^{du} \otimes \bar{\mathbf{S}}_{mm}^{du*} \right\rangle \simeq \left(\bar{\mathbf{F}}_m \otimes \bar{\mathbf{F}}_m^* \right) \left\langle \overset{...}{\bar{\mathbf{R}}}_{m-1,m} \otimes \overset{...}{\bar{\mathbf{R}}}_{m-1,m}^* \right\rangle \left\langle \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\} \otimes \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\}^* \right\rangle \left(\bar{\mathbf{F}}_m \otimes \bar{\mathbf{F}}_m^* \right) \tag{44a}$$

where we the two tensor products in the middle are assumed to be weakly correlated. A further approximation that we impose is given as follows

$$\left\langle \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\} \otimes \left\{ \bar{\mathbf{S}}_m^> + \bar{\mathbf{S}}_{mm}^{uu} \right\}^* \right\rangle \simeq \bar{\mathbf{S}}_m^> \otimes \bar{\mathbf{S}}_m^{>*} + \left\langle \bar{\mathbf{S}}_{mm}^{uu} \otimes \bar{\mathbf{S}}_{mm}^{uu*} \right\rangle. \tag{44b}$$

This is similar to the approximation we used while computing the Wigner transforms of the Green's functions. These are the kinds of approximations required to arrive at our boundary conditions.

5. Transition to Radiative Transfer

Now we have to transition from this transport equation (44) to the phenomenological radiative transfer equation discussed earlier. To accomplish this we have to link the key quantities of waves and radiative transfer, viz., coherence function and specific intensity. The relation between them is obtained by computing the energy

density using the two concepts. Thus we have

$$\frac{1}{2}\epsilon\left\{\langle|E_v(\mathbf{r})|^2\rangle + \langle|E_h(\mathbf{r})|^2\rangle\right\} = \frac{1}{c}\int d\Omega_s I(\mathbf{r},\hat{s}).\tag{45}$$

The Wigner transform provides us following relation:

$$\langle E_\mu(\mathbf{r})E_\nu^*(\mathbf{r})\rangle = \frac{1}{(2\pi)^2}\int d\mathbf{k}_\perp \mathcal{E}_{\mu\nu}(z,\mathbf{k}_\perp).\tag{46}$$

Defining $\mathcal{I}_{\mu\nu}$ as

$$\mathcal{I}_{\mu\nu}(z,\hat{s}) = \frac{1}{2\eta}\frac{k'^2}{(2\pi)^2}\cos\theta\mathcal{E}_{\mu\nu}(z,\mathbf{k}_\perp),\tag{47}$$

where η is the intrinsic impedance of the medium, we have from (45) and (46) $I = \mathcal{I}_{vv} + \mathcal{I}_{hh}$. To facilitate comparison with the results of Ulaby et al.[31], and Lam and Ishimaru [14] we will use a modified version of the Stokes vector [12]. Instead of the standard form $\{I,Q,U,V\}$ we use $\{(I+Q)/2,(I-Q)/2,U,V\}$. Thus, in terms of $\mathcal{I}_{\mu\nu}$ defined in (47) our modified Stokes vector is $\left\{\mathcal{I}_{vv},\mathcal{I}_{hh},\frac{1}{2}\left(\mathcal{I}_{vh}+\mathcal{I}_{hv}\right),-\frac{i}{2}\left(\mathcal{I}_{vh}-\mathcal{I}_{hv}\right)\right\}$.

There is still one difference that needs to be ironed out before we transition to the RT equations. Notice that in our wave approach we obtained transport equations for $\tilde{\mathcal{E}}$, which is the fluctuating part of the coherence function. On the other hand the phenomenological RT equations are traditionally written for total intensities. Therefore we have to express our transport equations in terms of $\mathcal{E}$. Notice that $\mathcal{E} = \bar{\mathcal{E}} + \tilde{\mathcal{E}}$, where $\bar{\mathcal{E}}$, the average part of $\mathcal{E}$ satisfies:

$$\left\{\frac{d}{dz} - ia(q_\mu - q_\nu^*)\right\}\bar{\mathcal{E}}_m^a(z,\mathbf{k}_\perp) = 0,\tag{48}$$

Using (48) in (39) we obtain

$$\left\{\frac{d}{dz} - i\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right]\right\}\mathcal{E}_{\mu\nu}^u(z,\mathbf{k}_\perp) = \frac{|k_m|^4}{(2\pi)^2}\int d\boldsymbol{\alpha}_\perp S_\mu^> S_\nu^{>*}(\boldsymbol{\mu}\cdot\boldsymbol{\mu}')(\boldsymbol{\nu}\cdot\boldsymbol{\nu}')\times$$

$$\times\Phi_m\left\{\mathbf{k}_\perp - \boldsymbol{\alpha}_\perp; \frac{1}{2}\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right] - \frac{1}{2}a\left[q_{\mu'}(\boldsymbol{\alpha}_\perp) - q_{\nu'}^*(\boldsymbol{\alpha}_\perp)\right]\right\}\mathcal{E}_{\mu'\nu'}^a(z,\boldsymbol{\alpha}_\perp),$$

$$\tag{49a}$$

$$\left\{-\frac{d}{dz} - i\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right]\right\}\mathcal{E}_{\mu\nu}^d(z,\mathbf{k}_\perp) = \frac{|k_m|^4}{(2\pi)^2}\int d\boldsymbol{\alpha}_\perp S_\mu^< S_\nu^{<*}(\boldsymbol{\mu}\cdot\boldsymbol{\mu}')(\boldsymbol{\nu}\cdot\boldsymbol{\nu}')\times$$

$$\times \Phi_m \left\{ \mathbf{k}_\perp - \boldsymbol{\alpha}_\perp; -\frac{1}{2}\left[q_\mu(\mathbf{k}_\perp) - q_\nu^*(\mathbf{k}_\perp)\right] - \frac{1}{2}a\left[q_{\mu'}(\boldsymbol{\alpha}_\perp) - q_{\nu'}^*(\boldsymbol{\alpha}_\perp)\right] \right\} \mathcal{E}_{\mu'\nu'}^a(z, \boldsymbol{\alpha}_\perp),$$

$$(49b)$$

Notice that this equation is expressed entirely in total intensity. Now we can transition to the phenomenological RT equations. Using the relation between $\mathcal{E}$ and I, we change the integration variable to solid angle and arrive at the following equation,

$$\left\{ \cos\theta \frac{d}{dz} + \gamma_{ij} \right\} I_j^u(z, \hat{s}) = \int d\Omega' P_{ij}^{ua}(\Omega, \Omega') I_j^a(z, \hat{s}'), \qquad (50a)$$

$$\left\{ -\cos\theta \frac{d}{dz} + \gamma_{ij} \right\} I_j^d(z, \hat{s}) = \int d\Omega' P_{ij}^{da}(\Omega, \Omega') I_j^a(z, \hat{s}'), \qquad (50b)$$

where $\bar{\bar{\gamma}}$ is the extinction matrix and $\bar{\bar{\mathbf{P}}}$ is the phase matrix. Implicit summation over superscripts a and subscript ν is assumed in (50). Although the structure of this equation is identical to that of the RT (equation (2)), the elements of the phase matrix and the extinction matrices are not the same primarily because of coherence induced by boundaries. As mentioned earlier, we assume that the layer thickness is greater than the mean free path of the corresponding medium. If, in addition, we assume the quasi-homogeneous field approximation we obtain the following expressions for the extinction and phase matrices:

$$\bar{\bar{\gamma}} = \cos\theta \ \text{diag} \ \{2q_v'', 2q_h'', q_v'' + q_h'', q_v'' + q_h''\} \qquad (51a)$$

$$P_{\mu\nu}^{ab} = \frac{1}{(2\pi)^2} \frac{1}{4} |k_m|^4 \Phi_m \left\{ \mathbf{k}_\perp - \mathbf{k}_\perp'; k_m'[a\cos\theta - b\cos\theta'] \right\} \mathcal{P}_{\mu\nu}^{ab}. \qquad (51b)$$

The double primes are used in (51a) to denote imaginary parts. For $\mu = v, h$,

$$\mathcal{P}_{\mu v}^{ab} = \left(\boldsymbol{\mu}^a \cdot \mathbf{v}'^b \right)^2 \qquad\qquad \mathcal{P}_{\mu h}^{ab} = \left(\boldsymbol{\mu}^a \cdot \mathbf{h}'^b \right)^2$$

$$\mathcal{P}_{\mu U}^{ab} = \left(\boldsymbol{\mu}^a \cdot \mathbf{v}'^b \right) \left(\boldsymbol{\mu}^a \cdot \mathbf{h}'^b \right) \qquad\qquad \mathcal{P}_{\mu V}^{ab} = 0 \qquad (52)$$

Similarly,

$$\mathcal{P}_{Uv}^{ab} = 2 \left(\mathbf{v}^a \cdot \mathbf{v}'^b \right) \left(\mathbf{h}^a \cdot \mathbf{v}'^b \right) \qquad \mathcal{P}_{Uh}^{ab} = 2 \left(\mathbf{v}^a \cdot \mathbf{h}'^b \right) \left(\mathbf{h}^a \cdot \mathbf{h}'^b \right)$$

$$\mathcal{P}_{UU}^{ab} = \left(\mathbf{v}^a \cdot \mathbf{v}'^b \right) \left(\mathbf{h}^a \cdot \mathbf{h}'^b \right) + \left(\mathbf{v}^a \cdot \mathbf{h}'^b \right) \left(\mathbf{h}^a \cdot \mathbf{v}'^b \right) \qquad \mathcal{P}_{UV}^{ab} = 0 \qquad (53)$$

$$\mathcal{P}_{Vv}^{ab} = \mathcal{P}_{Vh}^{ab} = \mathcal{P}_{VU}^{ab} = 0$$

$$\mathcal{P}_{VV}^{ab} = \left(\mathbf{v}^a \cdot \mathbf{v}'^b\right)\left(\mathbf{h}^a \cdot \mathbf{h}'^b\right) - \left(\mathbf{v}^a \cdot \mathbf{h}'^b\right)\left(\mathbf{h}^a \cdot \mathbf{v}'^b\right) \tag{54}$$

Noting the implied summation over a in (50) we see that they are identical to the RT equations given in Section 2. Now we have explicit expressions for the extinction matrix and phase matrix in terms of the statistical parameters of the problem thanks to our wave approach.

We next turn our attention to the boundary conditions (BC). In our wave approach we obtained BCs in terms of 'stack' reflection matrix $\overset{...}{\mathbf{R}}$, whereas in the RT approach the BCs are given in terms of the local interface reflection matrices. We can readily reconcile this apparent difference. Note that the BC in the wave approach forms a closed system whereas in the RT approach it is 'open' (linked to adjacent layer intensities). Let us take a look at the BC at the $(m-1)$-th interface. $\overset{...}{\mathbf{R}}_{m-2,m}$ can be expressed in terms of $\bar{\mathbf{R}}_{m-2,m-1}$ as follows,

$$\overset{...}{\mathbf{R}}_{m-1,m} = \bar{\mathbf{R}}_{m-1,m} + \bar{\mathbf{T}}_{m,m-1}\left\{I - \overset{...}{\mathbf{R}}_{m-2,m-1}\,\bar{\mathbf{F}}_{m-1}\bar{\mathbf{R}}_{m,m-1}\right\}^{-1}\overset{...}{\mathbf{R}}_{m-2,m-1}\,\bar{\mathbf{F}}_{m-1}\bar{\mathbf{T}}_{m-1,m}. \tag{55}$$

This is the relation between the stack reflection coefficients of adjacent interfaces. The $\bar{\mathbf{R}}$ and $\bar{\mathbf{T}}$ are local (single interface) reflection and transmission matrices at the $(m-1)$-th interface. On operating $\mathbf{E}_m^u$ with (55) we get

$$\mathbf{E}_m^d = \bar{\mathbf{R}}_{m-1,m}\mathbf{E}_m^u + \bar{\mathbf{T}}_{m,m-1}\mathbf{E}_{m-1}^d. \tag{56}$$

Notice that this boundary condition now involves only local interface Fresnel coefficients.

Similarly we write $\overset{...}{\mathbf{R}}_{m+1,m}$ in terms of $\overset{...}{\mathbf{R}}_{m+2,m+1}$ and hence obtain the BC at the m-th interface

$$\mathbf{E}_m^u = \bar{\mathbf{R}}_{m+1,m}\mathbf{E}_m^d + \bar{\mathbf{T}}_{m,m+1}\mathbf{E}_{m+1}^u. \tag{57}$$

Next we take the tensor product of (56) with its complex conjugate and average w.r.t. surface fluctuations. Employing the Wigner transform operator, we obtain the

boundary condition at the $(m-1)$-th interface, which is similar to that of the RT system. However, the reflection and transmission matrices used in the RT system correspond to the unperturbed medium as opposed to the average medium as in the case of the statistical wave approach. Similarly we can obtain the transport-theoretic boundary conditions at the m-th interface using (57).

6. Discussion

Now that we have made the transition from statistical wave theory to radiative transfer theory it is instructive to itemize the assumptions implicitly involved in the RT approach.

1. Quasi-stationary field approximation.

2. Weak fluctuations.

3. Statistical homogeneity of fluctuations.

These are the three well-known conditions necessary for the unbounded random medium problem.

However, if the medium is bounded we need to impose additional conditions. We found that the extinction coefficients calculated in the wave approach and the RT approach are different and only after applying further approximations can they be made to agree with each other. The following two additional conditions are required for our bounded random medium problem:

4. Layer thickness must be of the same order or greater than the corresponding mean free path.

5. Weak surface correlation approximation.

6. All fluctuations of the problem are statistically independent.

In this report we have modelled the random media as random continua. Another approach to this problem is the discrete random medium model [8, 15, 30, 12, 29, 19].

Recently Mishchenko et al. [19] (hereafter referred to as MTL for brevity) derived the vector radiative transfer equation (VRTE) for a bounded discrete random medium using a rigorous microphysical approach. This enabled them to identify the following assumptions embedded in the VRTE.

1. Scattering medium is illuminated by a plane wave.

2. Each particle is located in the far field zone of all other particles and the observation point is also located in the far field zones of all the particles forming the scattering medium.

3. Neglect all scattering paths going through a particle two or more times (Twersky approximation).

4. Assume that the scattering system is ergodic and averaging over time can be replaced by averaging over particle positions and states.

5. Assume that (i) the position and state of each particle are statistically independent of each other and those of all other particles and (ii) spatial distribution of the particles throughout the medium is random and statistically uniform.

6. Assume that the scattering medium is convex.

7. Assume that the number of particles N forming the scattering medium is very large.

8. Ignore all the diagrams with crossing connections in the diagrammatic expansion of the coherency dyadic.

It is apparent that there are distinct differences in the analyses for scattering from discrete and continuous random media. Hence it is not possible to make a one-to-one correspondence between the conditions of MTL and those in this report. Below is an attempt to make a connection between the two by considering each condition derived by MTL and relating it to ours. We will denote the condition numbers derived by MTL as MTL # and those obtained in this report as SM #.

MTL 1:- We also have a plane electromagnetic wave illuminating our system, although as pointed out by MTL it can be a quasi-plane wave.

MTL 2:- We also have implicitly employed the far field approximation. It is embedded in SM 1.

MTL 3:- This is embedded in SM 2. Although not explicitly stated, the scattering processes as mentioned in MTL 3 are avoided.

MTL 4:- In this report we have restricted our attention to the time-independent problem and hence did not encounter the issue of ergodicity.

MTL 5:- This condition is embedded in SM 3.

MTL 6:- In our problem we have distinct scattering boundaries and the character of the waves exiting or entering them are explicitly contained in the boundary conditions. Hence convexity of the scattering medium is not a necessary condition for us.

MTL 7:- This condition is embedded in SM 4.

MTL 8:- This condition is embedded in SM 2. Under weak fluctuation approximation we only take into consideration the leading term of the intensity operator.

Since the problem that we considered in this report involve scattering boundaries we have some additional conditions beyond those of MTL.

There are a few more remarks that we would like to make before closing. (a) In RT theory the medium is assumed to be sparse and hence the "refraction effects" of the fluctuations are ignored. Thus in the boundary conditions we should use the background medium parameters rather than the effective medium parameters as derived in our statistical wave theory. (b) To arrive at (50) from (49) we have ignored the contribution of evanescent modes. (c) The condition about statistical homogeneity of fluctuations may be relaxed by assuming it to be statistically quasi-homogeneous and we still can arrive at our results without much difficulty. (d) The

assumption regarding underlying statistics to be Gaussian is not only a convenience but also a reasonably good approximation in many applications. However, there are indeed certain situations where the statistics are not Gaussian. Similar analysis for such more general statistics are more complex and involved.

To summarize, we have enquired into the assumptions involved in adopting the radiative transfer approach to scattering from layered random media with rough interfaces. To facilitate this enquiry we adopted a wave approach to this problem and derived the governing equations for the first and second moments of the wave fields. We employed Wigner transforms and transitioned to the system corresponding to that of radiative transfer approach. In this process we found that there are more conditions implicitly involved in the RT approach to this problem than it is widely believed to be sufficient. With the recent development of fast and efficient algorithms for scattering computations and the enormous increase in computer resources it is now feasible to take an entirely numerical approach to this problem without imposing any approximations. In spite of such developments, to keep the size of the problem manageable only special cases have been studied thus far [11, 25, 22, 26]. Hence it is very much of relevance, interest and convenience to apply the RT approach to these problems. However, one should keep in mind the assumptions involved in such an approach. Otherwise interpretations of results based on RT theory can be misleading.

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